A Revised Qualitative Choice Logic for Handling Prioritized Preferences

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Abstract

Qualitative Choice Logic (QCL) is a convenient tool for representing and reasoning with "basic" preferences. However, this logic presents some limitations when dealing with complex preferences that, for instance, involve negated preferences. This paper proposes a new logic that correctly addresses QCL's limitations. It is particularly appropriate for handling prioritized preferences, which is very useful for aggregating preferences of users having different priority levels. Moreover, we show that any set of preferences, can equivalently be transformed into a set of normal form preferences from which efficient inferences can be applied.

1 Introduction

Decision analysis and Artificial Intelligence have been developed almost separately. Decision analysis is concerned with aggregation schemes and has relied mostly on numerical approaches, while Artificial Intelligence deals with reasoning and has an important logically oriented tradition [5]. Artificial Intelligence methods can contribute to a more implicit and compact representation of "agent's" preferences. This line of research has been recently illustrated in various ways by AI researchers [9, 7, 12, 2].
Recently, a new logic for representing choices and preferences has been proposed [1]. This logic, called *Qualitative Choice Logic (QCL)*, is an extension of propositional logic. The non-standard part of *QCL* logic is a new logical connective $\vec{\times}$, called *Ordered disjunction*,

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which is fully embedded in the logical language. Intuitively, if A and B are propositional formulas then A $\vec{\times}$ B means: "if possible A, but if A is impossible then at least B". As a consequence, *QCL* logic can be very useful to represent preferences for that framework. However, it presents some limitations. Assume that we want to represent the options concerning a travel from Paris to Vancouver. Assume that a travel agency has the following rules "customers preferring Air France to KLM also buy a hotel package" and "customers preferring KLM to Air France do not buy a hotel package". When a travel agency meets a customer that actually prefers Air France to KLM, the expected behavior of its information system is to propose a hotel package to that customer. Unfortunately, the *QCL* logic does not allow us to infer such a conclusion. It will infer both that a package should and should not be proposed.

In fact, the way negation is handled in *QCL* logic is not fully satisfactory. In *QCL* when a negation is used on a *QCL* formula with ordered disjunctions, that negated *QCL* formula is logically equivalent to a propositional formula obtained by replacing the ordered disjunction ($\vec{\times}$) by the propositional disjunction ($\vee$).

This is really a limitation, since for instance *QCL* does not make a distinction between the three rules: "Air France $\vec{\times}$ KLM $\Rightarrow$ FirstClass" (people preferring Air France to KLM travel in first class), "KLM $\vec{\times}$ Air France$\Rightarrow$FirstClass" (people preferring KLM to Air France travel in first class) and "Air France $\vee$ KLM $\Rightarrow$ FirstClass" (people flying on Air France or KLM travel in first class). In their two page short paper [4] these limitations of *QCL* have been informally advocated, however no rigorous solution is proposed.

This paper proposes a new logic called *PQCL* (Prioritized Qualitative Choice Logic). It is a new logic in the sense that negation, conjunction and disjunction departs from the ones used in standard *QCL*. However, it is based on the same *QCL* language. Our logic is dedicated for handling prioritized preferences and its inference relation correctly deals with negated preferences. In many applications, agent's preferences do not have the same level of importance. For instance, an agent who provides the two preferences : " I prefer AirFrance to KLM", and " I prefer a windows seat to a corridor seat", may consider that the first preference statement is more important that the second preference statement. Our logic can manage such prioritized preferences using prioritized conjunction, and disjunction.

One of the strong point of the *PQCL* logic proposed in this paper, is that its inference relation can be constructed in two equivalent ways. The first way is based on inference rules that define the satisfaction degree for any formula. The second way is based on a normal form function that equivalently transforms any set of preferences into a set of normal form preferences, from which efficient inferences can be applied. Indeed, having a set of preferences in a normal form allows us to reuse various non-monotonic approaches such as possibilistic logic [10] or compilation of stratified knowledge bases [16].

The rest of this paper is organized as follows. First, we recall *QCL* language, and we describe *QCL* logic limitations. Then, we introduce our new logic, called *PQCL*, that deals with prioritized preferences. We present the inference relation for our *PQCL* logic and show how $PQCL$ theories can be transformed into normal form theories. Last section concludes the paper.

2 The QCL language

This section presents the QCL language, which is in fact composed of three encapsulated sub-languages: Propositional Logic Language, the set of Basic Choice Formulas (BCF) or normal form preferences and the set of General Choice Formulas (GCF). These sub-languages are presented in the following subsections.

2.1 Basic Choice Formulas (BCF)

Let PS denotes a set of propositional symbols and $PROP_{PS}$ denotes the set of propositional formulas that can be built using classical logical connectives $(\Leftrightarrow, \Rightarrow, \wedge, \vee, \neg)$ over PS.

Basic choice formulas are ordered disjunctions of propositional formulas. They propose a simple way to order available alternatives. Given a set of propositional formulas $a_1, a_2, ..., a_n$, the formula $a_1 \vec{\times} a_2 \vec{\times} \ldots \vec{\times} a_n$[1] is used to express and ordered list of alternatives: some a_i must be true, preferably a_1, but if this is not possible then a_2, if this is not possible a_3, etc.

The language composed of *basic choice formulas* is denoted by BCF_{PS}, is the smallest set of words defined inductively as follow:

1. If $\phi \in PROP_{PS}$ then $\phi \in BCF_{PS}$
2. If $\phi, \psi \in BCF_{PS}$ then $(\phi \vec{\times} \psi) \in BCF_{PS}$
3. Every *basic choice formula* is only obtained by applying the two rules above a finite number of times.

BCF formulas represent simples alternatives between propositional formulas. In the rest of this paper BCF formulas are also called normal form formulas. The language of *basic choice formulas* has strong relationships with possibility theory, in particular with guaranteed possibility distributions, (see [1] for more details).

2.2 General Choice Formulas (GCF)

General Choice Formulas represent any formula that can be obtained from PS using connectors $\vec{\times}, \wedge, \vee, \neg$ on propositional formulas. The language composed of *general choice formulas*, denoted by QCL_{PS}, is defined inductively as follows:

1. If $\phi \in BCF_{PS}$ then $\phi \in QCL_{PS}$
2. If $\phi, \psi \in QCL_{PS}$ then $(\phi \wedge \psi), \neg(\psi), (\phi \vee \psi), (\phi \vec{\times} \psi) \in QCL_{PS}$.

[1] The operator $\vec{\times}$ is associative. Hence, $a_1 \vec{\times} a_2 \vec{\times} \ldots \vec{\times} a_n$ is used as a shorthand for $(((a_1 \vec{\times} a_2) \vec{\times} a_3) \ldots \vec{\times} a_n \ldots)))$.

3. The language of QCL_{PS} is only obtained by applying the two rules above a finite number of times.

GCF formulas represent the whole set of formulas that can be built using four connectives $(\vec{\times}, \wedge, \vee, \neg)$.

Example 1 *The formula "AirFrance $\vec{\times}$ KLM" is a Basic Choice Formula, while the formula "(AirFrance $\vec{\times}\neg$ KLM)$\vee$ (Class 1 $\vec{\times}$ Class 2)" is a General Choice Formula.*

For lake of space, we do not recall the inference process for QCL language. See [1] for a full description of QCL.

3 Limitations of QCL

As advocated in the introduction, the original QCL inference relation has a couple of (intuitively) undesirable properties. In the scope of a negation symbol or when occurring in the antecedent of a (material) implication, ordered disjunctions have not got their intended preferential reading. In fact, negated QCL formula is equivalent to plain propositional formula, obtained by replacing ordered disjunction by a standard disjunction, for instance :

$\neg(a_1 \vec{\times} a_2 \vec{\times} \ldots \vec{\times} a_n) \equiv \neg a_1 \wedge \neg a_2 \wedge \ldots \wedge \neg a_n$.

This means that, the double negation of any QCL formula is not equivalent to that formula, namely $\neg\neg(\phi)$ is not equivalent to ϕ. As a negative consequence, QCL does not make distinction between the following three rules :

1. $\neg$(AirFrance $\vec{\times}$ KLM) $\vee$ FirstClass,
2. $\neg$(KLM $\vec{\times}$ AirFrance) $\vee$ FirstClass,
3. $\neg$(AirFrance $\vee$ KLM) $\vee$ FirstClass.

They are all equivalent to the propositional formula ($\neg$AirFrance$\wedge\neg$KLM)$\vee$FirstClass.

4 Prioritized Qualitative Choice Logic *(PQCL)*

This section proposes a new logic called *Prioritized Qualitative Choice Logic (PQCL)*, which is characterized by new definitions of negation, conjunction and disjunction that are useful for aggregating preferences of users having different priority levels and overcome the QCL limitations. As in standard propositional logic, an interpretation I is an assignment of the classical truth values T,F to the atoms in PS. I will be represented by the set of its satisfied literals. The main features of our $PQCL$ logic are :

- The semantics of any formula is based on the degree of satisfaction of a formula in a particular model I. If an interpretation I satisfies a formula ϕ, then its satisfaction degree should be unique. We will use the notation $I \mid\!\sim_i^{PQCL} \phi$ to express that I satisfies ϕ to a degree i.

- Negation: Negation should be as a close as possible to the one of propositional logic. In particular, a double negation of a given formula should recover the original formula, namely we want $\neg(\neg\phi)$ to be equivalent to ϕ.
  Note that one cannot simply define $I|\!\sim_i \neg\phi$ iff "$I|\!\sim_i \phi$ is not true". Indeed, this implies that the satisfaction degree of a negated formula is not unique (which is not desirable). Namely, if one accepts $I|\!\sim_i \neg\phi$ iff $I|\!\sim_i \phi$ is not true, then if a given interpretation I satisfies ϕ to a degree 1 (namely, $I|\!\sim_1 \phi$), then this means that $I|\!\sim_2 \neg\phi$ and $I|\!\sim_3 \neg\phi$ are valid (since $I|\!\sim_2 \neg\phi$ and $I|\!\sim_3 \neg\phi$ are not valid), hence $\neg\phi$ is satisfied to different degrees which is not desirable.
  An additional feature is that the negation should be decomposable with respect to the conjunction and the disjunction, and it should satisfy De Morgan law.

- Prioritized preferences: Our *PQCL* logic should deal with prioritized preferences, encoded by means of a prioritized conjunction (resp. disjunction). For instance, an agent who provides the two preferences : " I prefer AF to KLM", and " I prefer a Windows seat to a Corridor seat" which we can formulate by "(AF $\vec{\times}$ KLM) $\wedge$(Windows$\vec{\times}$Corridor)" may consider that (AF $\vec{\times}$ Windows) is more important than (KLM $\vec{\times}$ Corridor).

4.1 The *PQCL* Inference relation

Before formally defining $|\!\sim^{PQCL}$, we need to introduce the notion of optionality, which is a revised version of the one given in [1].

Definition 1 (Optionality) *Let ϕ_1 and ϕ_2 be two formulas in QCL_{PS}. The optionality of a formulas is a function that assigns to each formula a strictly positive integer.*

- *opt(A) = 1 , A is a atom.*
- *opt($\phi_1 \vec{\times} \phi_2$)= opt($\phi_1$)+ opt($\phi_2$).*
- *opt($\phi_1 \wedge \phi_2$)= opt(ϕ_1)$\times$ opt(ϕ_2).*
- *opt($\phi_1 \vee \phi_2$)= opt(ϕ_1)$\times$ opt(ϕ_2).*
- *opt($\neg(\phi_1)$) = opt(ϕ_1).*

The optionality of a formula indicates the number of satisfaction degrees that a formula can have. The main difference with the original definition of optionality given in [1] concerns the three last definitions of optionality. In particular, in [1] opt($\neg\phi$) always equals 1 (since $\neg\phi$ is equivalent to a propositional formula). In our new logic $\neg\phi$ has the same optionality as ϕ. Note that the optionality of a propositional formulas is equal to 1.
The justification of optionality degree is directly related to the definition of of satisfaction degrees associated with interpretations. Let us explain, when dealing with prioritized preferences, why there are opt(ϕ_1)$\times$ opt(ϕ_2) options to satisfy ($\phi_1 \wedge \phi_2$). First, opt(ϕ_1)

(resp. opt(ϕ_2)) means that ϕ_1 (resp. ϕ_2) can be satisfied to a degree 1(first option of ϕ_1), to a degree 2(second option of ϕ_1),...,to a degree opt(ϕ_1)(the last option for ϕ_1), (resp. 1, 2,...,opt(ϕ_2)). Intuitively, given ($\phi_1 \wedge \phi_2$), the best and preferred solution is the one which satisfies the first option of ϕ_1 and the first option of ϕ_2. Then the second preferred solution is the one that still satisfies the first option of ϕ_1, but only satisfies the second option of ϕ_2. And more generally, an interpretation w is preferred to an interpretation w', if :

1. either w satisfies ϕ_1 to a degree i, and w' satisfies ϕ_1 to a degree j with j > i, or
2. both w and w' satisfy ϕ_1 to a same degree, but the degree on which ϕ_1 is satisfied in w is lower than the degree on which ϕ_2 is satisfied in w'.

Clearly, there are opt(ϕ_1)× opt(ϕ_2) options to satisfy $\phi_1 \wedge \phi_2$. And the worst solution is the one which satisfies ϕ_1 to a degree equal opt(ϕ_1) and ϕ_2 to a degree equal opt(ϕ_2). Given these optionality degrees associated with formulas, we are now able to define $\mid\!\sim^{PQCL}$. This is given by the following definition.

Definition 2 (The satisfaction relation) *Let ϕ_1, ϕ_2 be two formulas from QCL_{PS}. Let I be an interpretation. The following items give the definition of a satisfaction degree k of a formula $\phi_1$by I, denoted by $I\mid\!\sim^{PQCL}_k \phi_1$.*

1. $I\mid\!\sim^{PQCL}_k a$ iff $k = 1$ and $a \in I$ (for propositional atoms a).
2. $I\mid\!\sim^{WQCL}_k \neg a$ iff $k = 1$ and $\neg\ a \in I$ (for propositional atoms a).
3. $I\mid\!\sim^{PQCL}_k (\phi_1 \vec{\times} \phi_2)$ iff *(*$I\mid\!\sim^{PQCL}_k \phi_1$*)* or *(*$I\mid\!\sim^{PQCL}_n \phi_2$ and there is no m such that $I\mid\!\sim^{PQCL}_m \phi_1$, and $k = n + opt(\phi_1)$*)*.
4. $I\mid\!\sim^{PQCL}_k (\phi_1 \vee \phi_2)$ iff one of the following cases is satisfied *:*
   (a) *(*$I\mid\!\sim^{PQCL}_1 \phi_1$*)* or *(*$I\mid\!\sim^{PQCL}_1 \phi_2$*)* and k = 1.
   (b) *(There exists $i > 1$,* $I\mid\!\sim^{PQCL}_i \phi_1$*)* and [$\nexists$ m such that $I \mid\!\sim^{PQCL}_m \phi_2$], and k = (i-1) × opt($\phi_2$) + 1.
   (c) *(There exists $i > 1$ such that $I\mid\!\sim^{PQCL}_i \phi_1$ or $\nexists$ l, such that $I\mid\!\sim^{PQCL}_l \phi_1$)* and *(there is $j > 1$, $I\mid\!\sim^{PQCL}_j \phi_2$)*, and $k = j$.
5. $I\mid\!\sim^{PQCL}_k (\phi_1 \wedge \phi_2)$ iff $I\mid\!\sim^{PQCL}_i (\phi_1)$ and $I\mid\!\sim^{PQCL}_j (\phi_2)$ and $k = (i\text{-}1) \times opt(\phi_2) + j$.
6. $I\mid\!\sim^{PQCL}_k \neg(\phi_1 \vee \phi_2)$ iff $I\mid\!\sim^{PQCL}_k \neg\phi_1 \wedge \neg\phi_2$.
7. $I\mid\!\sim^{PQCL}_k \neg(\phi_1 \wedge \phi_2)$ iff $I\mid\!\sim^{PQCL}_k \neg\phi_1 \vee \neg\phi_2$.
8. $I\mid\!\sim^{PQCL}_k \neg(\phi_1 \vec{\times} \phi_2)$ iff $I\mid\!\sim^{PQCL}_k \neg\phi_1 \vec{\times} \neg\phi_2$.

9. $I{\mid\!\sim}_k^{PQCL}\neg(\neg\phi_1)$ iff $I{\mid\!\sim}_k^{PQCL}\phi_1$.

Let us explain the definition of our inference relation. Items (2), (6), (7), (8), (9) deals with the satisfaction of negated formulas. They simply say that the negation is decomposable, and satisfies De Morgan rule. Items (5) and (6) deals with the satisfaction of conjunction and the satisfaction of disjunction. Clearly, they deal with prioritized preferences. The way the degree is defined on $\phi_1 \wedge \phi_2$ reflects some lexicographical ordering between individual satisfaction degree of ϕ_1 and ϕ_2. Namely, given two interpretations I, I' then I is strictly preferred to I', if :

1. either $I{\mid\!\sim}_i\phi_1$ and $I'{\mid\!\sim}_j\phi_1$ with $i < j$,
2. or ($I{\mid\!\sim}_i\phi_1$ and $I'{\mid\!\sim}_i\phi_1$) and ($I{\mid\!\sim}_k\phi_2$ and $I'{\mid\!\sim}_l\phi_2$ with $k < l$).

Example 2 *Let us illustrate the inference relation of* PQCL *by the following example. Let* $\phi = (AF\vec{\times}KLM) \wedge (W\vec{\times}C)$, *and* $I = \{KLM, C\}$. *The formula* ϕ *is of the form* $(\phi_1 \wedge \phi_2)$ *with* $\phi_1 = (AF\vec{\times}KLM)$ *and* $\phi_2 = W\vec{\times}C$.
We have $I{\mid\!\sim}_{i=2}^{PQCL}\phi_1$ *and* $I{\mid\!\sim}_{j=2}^{PQCL}\phi_2$ *(by using item (3) of Definition 2). Thus, applying item (5) of Definition 2, we obtain* $I{\mid\!\sim}_k^{PQCL}\phi$ *and* $k = (i\text{-}1)\times opt(\phi_2) + j = 4$.

Comparing to the original QCL inference relation defined in [1], only the item (1) and the item (3) are the same. All others are different. Namely, our definition of negation, conjunction and disjunction are completely different. However, there is situation where QCL and $PQCL$ collapse. It is when restricting to propositional or basic choice formulas, more precisely :

Proposition 1 *1. Let* ϕ *be a propositional formula, and* I *an interpretation, then :* $I{\mid\!\sim}_1^{QCL}\phi$ *iff* $I{\mid\!\sim}_1^{PQCL}\phi$ *iff* $I \models \phi$.

2. Let $\phi = a_1\vec{\times}a_2\vec{\times}\ldots\vec{\times}a_n$ *be a basic choice formula namely,* a_i *are propositional formulas, then :* $I{\mid\!\sim}_k^{QCL}\phi$ *iff* $I{\mid\!\sim}_k^{PQCL}\phi$ *iff* $I \models a_1 \vee a_2 \vee ... \vee a_n$ *and* $k = min\{j \mid I \models a_j\}$.

Namely, a basic choice formula $a_1\vec{\times}a_2\vec{\times}\ldots\vec{\times}a_n$ is satisfied to a degree k by an interpretation I if I satisfies a_k but fails to satisfy a_i for all $1 \leq i < k$.

The above proposition shows that our definitions of disjunction and conjunction extend the ones of classical logic when they are applied to propositional formulas. But of course they are non-classical since they can be used on non-propositional (general QCL) formulas. Now, we define an inference relation between a $PQCL$ theory and a propositional formula. It follows the same steps defined in [1]. Let K be a set of propositional formulas which represents knowledge or integrity constraints, and let T be a set of preferences. We need to define the notion of preferred models.

Definition 3 *Let* $M^k(T)$ *denote the subset of formulas of* T *satisfied by a model* M *to a degree* k. *A model* M_1 *is* $K \cup T$*-preferred over a model* M_2 *if there is a* k *such that* $\mid M_1^k(T)\mid > \mid M_2^k(T)\mid$ *and for all* $j < k$: $\mid M_1^j(T)\mid = \mid M_2^j(T)\mid$.
M *is a preferred model of* $K \cup T$ *iff :*

1. *M is model of K, and*

2. *M is maximally $(K \cup T)$-preferred.*

The following definition gives the inference relation between ($K \cup T$) and a propositional formula ϕ.

Definition 4 *Let K be a set of formulas in $PROP_{PS}$ and T be a set of formulas in GCF_{PS}, and ϕ be a formula in $PROP_{PS}$.*
$K \cup T \mid\!\sim_k^{PQCL} \phi$ iff ϕ is satisfied in all preferred models of $K \cup T$.

The following contains an example which shows that $PQCL$ overcomes one the limitations of QCL advocated in the introduction. Note that if one wants to express that ϕ_1, ϕ_2 are equally really, then it is en-ought to separately put them in T.

Example 3 *Let us consider the example given in the introduction where our knowledge base K contains $\neg KLM \vee \neg AirFrance$* **(1)** *and T contains the following preferences:*

$$\begin{cases} \neg(AirFrance \vec{\times} KLM) \vee HotelPackage \quad (\mathbf{2}) \\ \neg(KLM \vec{\times} AirFrance) \vee \neg HotelPackage \quad (\mathbf{3}) \\ AirFrance \vec{\times} KLM \quad (\mathbf{4}) \end{cases}$$

To give the preferred model of $K \cup T$, we should firstly give the satisfaction degree of the formulas **(1)**, **(2)**, **(3)** *and* **(4)** *for each interpretation, so for instance, by using Definition 2. Let $I = \{AirFrance, hotelpackage\}$, We have $I \models \neg KLM \vee \neg AirFrance$, $I \mid\!\sim_1^{PQCL} AirFrance \vec{\times} KLM$ (using Proposition 1).*
Consider now the preference **(2)**, *namely $\neg(AirFrance \vec{\times} KLM) \vee HotelPackage$. Using item (8) of Definition 2, we get $I \mid\!\sim_2^{PQCL} \neg(AirFrance \vec{\times} KLM)$, and $I \mid\!\sim_1^{PQCL} HotelPackage$. Using item (4)-a of Definition 2, we get $I \mid\!\sim_1^{PQCL} (\neg AirFrance \vec{\times} KLM) \vee HotelPackage$. Similarly, we also have $I \mid\!\sim_1^{PQCL} \neg(KLM \vec{\times} AirFrance) \vee \neg HotelPackage$, since $I \mid\!\sim_1^{PQCL} \neg(KLM \vec{\times} AirFrance)$.*
The following truth table summarizes for each formula **(1)**, **(2)**, **(3)**, **(4)** *from K and T we are interested in, whether it is satisfied (T) (to some degree) or not (-) by a given interpretation.*

AirFrance	*KLM*	*Hotelpackage*	**(1)**	**(2)**	**(3)**	**(4)**
F	*F*	*F*	*1*	*1*	*1*	-
F	*F*	*T*	*1*	*1*	*1*	-
F	*T*	*F*	*1*	*1*	*1*	*2*
F	*T*	*T*	*1*	*1*	*2*	*2*
T	*F*	*F*	*1*	*2*	*1*	*1*
T	**F**	**T**	**1**	**1**	**1**	**1**
T	*T*	*F*	-	-	*1*	*1*
T	*T*	*T*	-	*1*	-	*1*

Table 1: The models of $K \cup T$ by using $PQCL$.

In original QCL logic, $K \cup T$ is declared to be inconsistent. With our PQCL logic, $K \cup T$ has one preferred model (bold line), $I = \{AirFrance, Hotelpackage\}$, from which we obtain the expected conclusion $K \cup T \mid\!\sim^{PQCL} Hotelpackage$.

5 Normalization form

In this section, we show that any set of preferences, can equivalently be transformed into a set of normal form preferences, which are simply basic choice formulas.
We need to introduce the notion of equivalence between two formulas in QCL_{PS}. It is given by the following definition:

Definition 5 *Two QCL_{PS} formulas ϕ_1 and ϕ_2 are said to be equivalent, denoted simply by $\phi_1 \equiv \phi_2$, if:*

- *For all interpretation I, and integer k we have $I \mid\!\sim_k^{PQCL} \phi_1$ iff $I \mid\!\sim_k^{PQCL} \phi_2$.*
- *$opt(\phi_1) = opt(\phi_2)$.*

The following introduces a normal form function, which associates with each general choice formula, its corresponding basic choice formula. This normal form function, denoted by $\mathcal{N}$, will allow us to transform any set of preferences into a set of basic choice formulas. This is very important from computation point of view, since it allow us to reuse various non-monotonic approaches such as possibilistic logic [10] or compilation of stratified knowledge bases [16].
Let ϕ be a formula in QCL_{PS}, let $\phi_1 = a_1 \vec{\times} a_2 \vec{\times} \ldots \vec{\times} a_n$, and $\phi_2 = b_1 \vec{\times} b_2 \vec{\times} \ldots \vec{\times} b_m$ be two basic choice formulas. The idea in defining the normal form of ϕ, denoted by $\mathcal{N}(\phi)$, is very simple. If ϕ is of the forme $\phi_1 \vec{\times} \phi_2$ (resp. $\neg\phi_1$, $\phi_1 \wedge \phi_2$, $\phi_1 \vee \phi_2$) then $\mathcal{N}(\phi)$ is simply the result of applying $\vec{\times}$ (resp. $\neg$, $\wedge$, $\vee$) to the normal form of its components. Namely:

1. $\mathcal{N}(\neg\phi_1) \equiv \mathcal{N}(\neg\mathcal{N}(\phi_1))$.
2. $\mathcal{N}(\phi_1 \wedge \phi_2) \equiv \mathcal{N}(\mathcal{N}(\phi_1) \wedge \mathcal{N}(\phi_2))$.
3. $\mathcal{N}(\phi_1 \vee \phi_2) \equiv \mathcal{N}(\mathcal{N}(\phi_1) \vee \mathcal{N}(\phi_2))$.
4. $\mathcal{N}(\phi_1 \vec{\times} \phi_2) \equiv \mathcal{N}(\mathcal{N}(\phi_1) \vec{\times} \mathcal{N}(\phi_2))$.
   If ϕ is already a basic choice formula, then $\mathcal{N}(\phi)$ is always equal to ϕ, namely
5. $\forall \phi_1 \in BCF_{PS}, \mathcal{N}(\phi_1) = \phi_1$.
   Hence, it only remains to define $\mathcal{N}(\phi_1 \wedge \phi_2)$ (resp. $\mathcal{N}(\phi_1 \vee \phi_2)$, $\mathcal{N}(\neg\phi_1)$) where ϕ_1 and ϕ_2 are normal form. This is given by items 6, 7, 8. Namely:
6. $\mathcal{N}(\phi_1 \wedge \phi_2) \equiv c_{11} \vec{\times} \ldots \vec{\times} c_{1m} \vec{\times} c_{21} \vec{\times} \ldots \vec{\times} c_{2m} \vec{\times} c_{n1} \vec{\times} \ldots \vec{\times} c_{nm}$, with $c_{ij} = a_i \wedge b_j$.

7. $\mathcal{N}(\phi_1 \vee \phi_2) \equiv d_{11} \vec{\times} \ldots \vec{\times} d_{1m} \vec{\times} d_{21} \vec{\times} \ldots \vec{\times} d_{2m} \vec{\times} d_{n1} \vec{\times} \ldots \vec{\times} d_{nm}$, with $d_{ij} = a_i \vee b_j$.

8. $\mathcal{N}(\neg\phi_1) \equiv \neg a_1 \vec{\times} \neg a_2 \vec{\times} \ldots \vec{\times} \neg a_n$.

Property 6 confirms the meaning of prioritized conjunction. Indeed, assume $a_1 \vec{\times} a_2 \vec{\times} \ldots \vec{\times} a_n$ denotes a preference of a user A, and $b_1 \vec{\times} b_2 \vec{\times} \ldots \vec{\times} b_m$ denotes a preference of B. Applying, prioritized conjunction allows to select solutions that privileges A. For instance, $a_1 b_m$ (which represents the best choice for A and the worst choice for B) is preferred to $a_2 b_1$ (which represents the best choice for B and the second choice for A).

Proposition 2 *Let K be a set of propositional formulas and T be a set of general choice formulas. Let T' be a set of basic choice preferences obtained from T by replacing each ϕ in T by $\mathcal{N}(\phi)$, then $\forall \phi$, $K \cup T \mid\!\sim^{PQCL} \phi$ iff $K \cup T' \mid\!\sim^{PQCL} \mathcal{N}(\phi)$.*

A sketch of proof is provided in the appendix.
As a consequence, for any formula in QCL_{PS}, we have two possibilities to implement it:

1. Using a relation of satisfaction on general choice formulas, as indicated in Definition 2, or

2. Normalize or generate a set of basic choice formulas from any set of preferences, we then apply the inference relation from BCF theories as indicated in Proposition 3,

Example 4 *Let us illustrate Proposition 7 by the following example. Let $\phi = ((a \vec{\times} b) \vee \neg c) \wedge (c \vec{\times} b)$, and $I = \{b,c\}$. We use two different ways to give the satisfaction degree of this formula.*

1. *We normalize the set of preferences into a set of normal form preferences :*
   The formula $((a \vec{\times} b) \vee \neg c) \wedge (c \vec{\times} b)$ is a general choice formula, thus using the normal form function, we have
   $\mathcal{N}(((a \vec{\times} b) \vee \neg c) \wedge (c \vec{\times} b)) \equiv \mathcal{N}(\mathcal{N}((a \vec{\times} b) \vee \neg c) \wedge \mathcal{N}(c \vec{\times} b))$,
   $\equiv \mathcal{N}([(a \vee \neg c) \vec{\times} (b \vee \neg c)] \wedge (c \vec{\times} b))$,
   $\equiv \mathcal{N}([(a \vee \neg c) \wedge c] \vec{\times} [(a \vee \neg c) \wedge b] \vec{\times} [(b \vee \neg c) \wedge c] \vec{\times} [(b \vee \neg c) \wedge b])$,
   $\equiv (a \wedge c) \vec{\times} ((a \vee \neg c) \wedge b) \vec{\times} (b \wedge c) \vec{\times} ((b \vee \neg c) \wedge b)$
   The obtained formula is a basic choice formula, using Proposition 3, we have $I \not\models (a \wedge c)$ and $I \not\models ((a \vee \neg c) \wedge b)$ but $I \models (b \wedge c)$, thus $I \mid\!\sim_3^{PQCL} \phi$.

2. *Now, we use directly Definition 2 :*
   The formula ϕ is of the form $(\phi_1 \wedge \phi_2)$ with $\phi_1 = (a \vec{\times} b) \vee \neg c$ and $\phi_2 = c \vec{\times} b$.
   We have $I \mid\!\sim_{j=1}^{PQCL} \phi_2$. The formula ϕ_1 is of the form $\phi' \vee \phi''$ with $\phi' = a \vec{\times} b$ and $\phi'' = \neg c$. So we have $I \mid\!\sim_{i'=2}^{PQCL} a \vec{\times} b$ and $I \not\models \neg c$, then $I \mid\!\sim_i^{PQCL} \phi_1$ and $i = (i'\text{-}1) \times opt(\phi'') + 1 = (2\text{-}1) \times 1 + 1 = 2$. Lastly, we apply item (5) of Definition 2 we obtain, $I \mid\!\sim_k^{PQCL} \phi$ and $k = (i\text{-}1) \times opt(\phi_2) + j = 3$. Hence, we get the same result.

6 Conclusions

The problem of representing preferences has drawn attention from Artificial Intelligence researchers. The paper on preference logic [17] addresses the issue of capturing the common-sense meaning of preference through appropriate axiomatizations. The papers on preference reasoning [11, 12, 6] attempt to develop practical mechanisms for making inference about preferences and making decisions. A principal concept there is Ceteris Paribus preference: preferring one outcome to another, everything else being equal. The work on prioritized logic programming and non-monotonic reasoning [3, 14, 15] has potential applications to databases. CP-nets [6, 8] use Bayesian-like structure to represent preferences under again Ceteris Paribus principle. However, in the majority of theses works, the negation in the context of representing preferences is not discussed, and few of them integrated prioritized preferences.

In this paper, a new logic for representing prioritized preferences has been proposed. This logic is characterized by new definitions of negation, conjunction and disjunction that are useful for aggregating preferences of users having different priority levels and overcome the QCL limitations. It generalizes the way the inference is done by presenting an inference framework based on normal form function or directly by using the satisfaction relation.

A future work is to apply our methods to alarms filtering. Indeed, existing alerts correlations and alerts filtering do not use take into account administrator's preferences. Our PQCL will be basically use to compactly represent administrator's preferences, and will be use to rank-order alerts to be presented to netwok administrator.

Appendix. Proof of proposition 7

For sake of space, we can only give the sketch of proof of the conjunction and disjunction. Let $\phi_1 = a_1 \vec{\times} a_2 \vec{\times} \ldots \vec{\times} a_n$, and $\phi_2 = b_1 \vec{\times} b_2 \vec{\times} \ldots \vec{\times} b_m$.

1. Let us give the proof of $I \mid\!\sim_k^{PQCL} (\phi_1 \wedge \phi_2)$ iff $I \models_k \mathcal{N}(\phi_1 \wedge \phi_2)$.
   Recall that : $\mathcal{N}(\phi_1 \wedge \phi_2) \equiv\ c_{11} \vec{\times} \ldots \vec{\times}\ c_{1m} \vec{\times}\ c_{21} \vec{\times} \ldots \vec{\times}\ c_{2m} \vec{\times} \ldots \vec{\times}\ c_{n1} \vec{\times} \ldots \vec{\times}\ c_{nm}$, with $c_{ij} = a_i \wedge b_j$. Let us consider different cases of satisfaction of $a_i{}'s$ and $b_i{}'s$ by the interpretation I.

   - Suppose that there exists i > 0 and j > 0 such that $I \models_i\ a_1 \vec{\times} \ldots \vec{\times}\ a_n$ and $I \models_j\ b_1 \vec{\times} \ldots \vec{\times}\ b_m$, this also means that $I \models \neg a_1 \wedge ... \wedge \neg a_{i-1} \wedge a_i$ and $I \models \neg b_1 \wedge ... \wedge \neg b_{j-1} \wedge b_j$.
     This means that I falsifies $\{ c_{11}, \ldots, c_{1m}, c_{21}, \ldots, c_{2m}, \ldots, c_{i1}, \ldots, c_{i(j-1)} \}$, but I satisfies $c_{ij} (= a_i \wedge b_j)$. We have (i-1)×m + j -1 items which are not satisfied before satisfying $(a_i \wedge b_j)$. So, this means that $I \models_k\ c_{11} \vec{\times} \ldots \vec{\times}\ c_{1m} \vec{\times}\ c_{21} \vec{\times} \ldots \vec{\times}\ c_{2m} \vec{\times}\ c_{n1} \vec{\times} \ldots \vec{\times}\ c_{nm}$, namely $I \models_k \phi_1 \wedge \phi_2$ and k = (i-1)×m + j.
     Hence using item (5) of Definition 2, we can check that we also have $K \cup T \mid\!\sim_k^{PQCL} \phi_1 \wedge \phi_2$ and k= (i-1)×opt(ϕ_2) + j.
   - There is no i such that $I \models_i\ a_1 \vec{\times} \ldots \vec{\times}\ a_n$ or there is no j such that $I \models_j$

$b_1\vec{\times}\ldots\vec{\times}\ b_m$. This means that I either falsifies all $a_i's$, or I falsifies all $b_j's$. Thus I falsifies c_{ij} ($= a_i \wedge b_j$), namely there is no k such that $I \models_k (\phi_1 \wedge \phi_2)$. Hence $K \cup T \not\mid\!\sim^{PQCL} \phi_1 \wedge \phi_2$.

2. Let us give the proof of $I \mid\!\sim_k^{PQCL} (\phi_1 \vee \phi_2)$ iff $I \models_k \mathcal{N}(\phi_1 \vee \phi_2)$.
   Recall that : $\mathcal{N}(\phi_1 \vee \phi_2) \equiv\ d_{11}\vec{\times}\ldots\vec{\times}\ d_{1m}\vec{\times}\ d_{21}\vec{\times}\ldots\vec{\times}\ d_{2m}\vec{\times}\ldots\vec{\times}\ d_{n1}$ $\vec{\times}\ldots\vec{\times}\ d_{nm}$, with $d_{ij} =\ a_i \vee\ b_j$. Let us consider different cases of satisfaction of $a_i's$ and $b_i's$ by the interpretation I.

   - Suppose that there exists i > 0 or j > 0 such that $I \models_i\ a_1\vec{\times}\ldots\vec{\times}\ a_n$ and $I \models_j\ b_1\vec{\times}\ldots\vec{\times}\ b_m$. This means that there exists k > 0 such that $I \models_k\ d_k$. We can distinguish different cases :
     - If i = 1 or j = 1, then I satisfies { $d_{11},\ d_{12}, \ldots,\ d_{1(j-1)},\ d_{1j}, \ldots,\ d_{1m}$, $d_{21}, \ldots d_{i1}$} but I falsifies the rest of items. In this case, the first satisfied item is d_{11}, thus $I \models_1\ d_{11}\vec{\times}\ldots\vec{\times}\ d_{1m}\vec{\times}\ d_{21}\vec{\times}\ldots\vec{\times}\ d_{2m}\vec{\times}\ \ldots\vec{\times}\ d_{n1}\vec{\times}\ldots\vec{\times}\ d_{nm}$. Hence $I \models_1 (\phi_1 \vee \phi_2)$.
       Using item (4)-a of Definition 2, we can check that we also have $K \cup T \mid\!\sim_k^{PQCL} \phi_1 \vee \phi_2$ and k =1.
     - If i > 1 or j > 0, then I falsifies { $d_{11},\ d_{12}, \ldots,\ d_{1(j-1)},\ d_{21}, \ldots,\ d_{2(j-1)}, \ldots d_{i1}, \ldots, d_{i(j-1)}$} but I satisfies { d_{1j},..., $d_{2j}, \ldots,\ d_{ij}$}.
       In this case, we have (j-1) not satisfied items before the first satisfied item d_{1j}, thus $I \models_j\ d_{11}\vec{\times}\ldots\vec{\times}\ d_{1m}\vec{\times}\ d_{21}\vec{\times}\ldots\vec{\times}\ d_{2m}\vec{\times}\ \ldots\vec{\times}\ d_{n1}\vec{\times}\ldots\vec{\times}\ d_{nm}$. Hence $I \models_j (\phi_1 \vee \phi_2)$.
       Using item (4)-c of Definition 2, we have $K \cup T \mid\!\sim_k^{PQCL} \phi_1 \vee \phi_2$ and k =j.
   - There is no i such that $I \models_i\ a_1\vec{\times}\ldots\vec{\times}\ a_n$ and there is j such that $I \models_j$ $b_1\vec{\times}\ldots\vec{\times}\ b_m$. This means that $I \models \neg a_1 \wedge \ldots \wedge \neg a_{i-1} \wedge \neg a_i \vee \ldots \vee \neg a_n$ and $\exists$ j > 0 such that $I \models \neg b_1 \wedge \ldots \wedge \neg b_{j-1} \wedge b_j$.
     This also means that I falsifies { $d_{11}, \ldots,\ d_{1(j-1)}, \ldots,\ d_{21}, \ldots,\ d_{2(j-1)}\ \ldots,\ d_{(i-1)(j-1)}$}, but I satisfies the items {d_{1j}, d_{2j},..., $d_{(i-1)j}$, d_{ij}}. So, d_{1j} is the first satisfied item and all the items before d_{1j} are not satisfied, namely we have at least (j-1) not satisfied items before the first satisfied item ($a_1 \vee b_j$), this means that $I \models_k\ d_{11}\vec{\times}\ldots\vec{\times}\ d_{1m}\vec{\times}\ d_{21}\vec{\times}\ldots\vec{\times}\ d_{2m}\vec{\times}\ \ldots\vec{\times}\ d_{n1}\vec{\times}\ldots\vec{\times}\ d_{nm}$, hence $I \models_j (\phi_1 \vee \phi_2)$.
     Using item (4)-c of Definition 2, we have also $K \cup T \mid\!\sim_k^{PQCL} (\phi_1 \vee \phi_2)$, and k = j.
   - There is i such that $I \models_i\ a_1\vec{\times}\ldots\vec{\times}\ a_n$ and there is no j such that $I \models_j$ $b_1\vec{\times}\ldots\vec{\times}\ b_m$. This means that there $\exists$ i > 0 such that $I \models \neg a_1$ $wedge \ldots \wedge \neg a_{i-1} \wedge a_i$ and $\nexists$ j such that $I \models \neg b_1 \wedge \ldots \wedge \neg b_{j-1} \wedge \neg b_j$.
     This means that I falsifies { $d_{11}, \ldots,\ d_{1(j-1)}, d_{1j} \ldots,\ d_{21}, \ldots,\ d_{2(j-1)}, d_{2j}$,

..., $d_{(i-1)(j-1)}\}$, but I satisfies the items $\{d_{i1}$, ..., $d_{(i)(j-1)}$, $d_{ij}\}$. So, d_{i1} is the first satisfied item and all the items before d_{i1} $(=a_i \vee b_1)$ are not satisfied, namely we have at least (i-1)×m not satisfied items, this means that $I \models_k d_{11} \vec{\times} \ldots \vec{\times}\ d_{1m} \vec{\times}\ d_{21} \vec{\times} \ldots \vec{\times}\ d_{2m} \vec{\times} \ldots \vec{\times}\ d_{n1} \vec{\times} \ldots \vec{\times}\ d_{nm}$, so $I \models_k (\phi_1 \vee \phi_2)$, and k = (i-1)×m +1.
Hence using item (4)-b of Definition 2, we have also $K \cup T {\mid\!\sim}_k^{PQCL} (\phi_1 \vee \phi_2)$, and k = (i-1)×opt($\phi_2$) +1.

- There is no i such that $I \models_i\ a_1 \vec{\times} \ldots \vec{\times}\ a_n$ and there is no j such that $I \models_j b_1 \vec{\times} \ldots \vec{\times}\ b_m$. This means that I either falsifies all $a_i's$, I falsifies all $b_j's$. Hence I falsifies all d_{ij} $(=\ a_i \vee\ b_j)$.
Hence, there is no k such that $I \models_k (\phi_1 \vee \phi_2)$, so $K \cup T \not{\mid\!\sim}^{PQCL} (\phi_1 \vee \phi_2)$.

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